Radiation-induced resistance oscillations in a 2D hole gas: a demonstration of a universal effect.

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We report on a theoretical insight about the microwave-induced resistance oscillations and zero resistance states when dealing with p-type semiconductors and holes instead of electrons. We consider a high-mobility two-dimensional hole gas hosted in a pure Ge/SiGe quantum well. Similarly to electrons we obtain radiation-induce resistance oscillations and zero resistance states. We analytically deduce a universal expression for the irradiated magnetoresistance, explaining the origin of the minima positions and their 1/4 cycle phase shift. The outcome is that these phenomena are universal and only depend on radiation and cyclotron frequencies. We also study the possibility of having simultaneously two different carriers driven by radiation: light and heavy holes. As a result the calculated magnetoresistance reveals an interference profile due to the different effective masses of the two types of carriers.

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I. INTRODUCTION

High-mobility two-dimensional electron systems (2DES) are fantastic platforms for studying transport and coupling with different potentials, static or time-dependent (radiation) in nano-systems. In the last decade two of the most striking effects involving radiation-matter coupling in 2DES were discovered: the microwave-induced resistance oscillations (MIRO) and zero resistance states (ZRS)[1,2]. They are indeed remarkable effects that surprised condensed matter community when they were discovered. Mainly because they involve simultaneously radiation-matter interaction[3] and transport excited by radiation in a nanoscopic system. On the other hand their discovery was considered also very important, specially in the case of zero resistance states, because they were obtained without quantization in the Hall resistance. The interest in both effects is focussed not only on the basic explanation of a physical effect but also on its potential applications. They are obtained when 2DES, in high mobility samples at low temperature ($\sim 1K$), are subjected to a perpendicular magnetic field (B) and radiation (microwave (MW) band) simultaneously. In these experiments, for an increasing radiation power (P), one first obtains longitudinal magnetoresistance (R_{xx}) oscillations which turn into zero resistance states (ZRS) at high enough P.

Many experiments have been carried out[4–17] and theoretical explanations[18–24] have been given to try to explain their physical origin, but to date, it still remains controversial. On the other hand, although these effects have been thoroughly studied, they have been mainly based on GaAs/AlGaAs quantum wells and hardly ever other materials have been used. Yet, we can cite interesting experimental results about radiation-induced magnetotransport oscillations on a different non-degenerate 2D system such as electrons on liquid helium surface[13,14]; they may share a similar physical origin as MIRO. In this way we wonder if MIRO and ZRS are universal effects and if, as a result, they can be observed in different platforms. For instance, different materials and carriers such as holes working with valence bands in p-type materials[25]. In contrast to 2DES, a two-dimensional hole gas (2DHG) presents more non-linearities and a more interesting dynamics when it comes to coupling with radiation.

In this article, we demonstrate that these effects are universal phenomena and that they can be obtained as well in a 2DHG. Based on previous results[18,26–28] we obtain a universal expression for irradiated R_{xx}. According to it, MIRO only depend on radiation and cyclotron frequencies and not on the type of semiconductor material. We are able to explain the experimentally obtained MIRO extrema and node positions; the 1/4 cycle phase shift of MIRO minima. We have applied the results to the case of holes obtaining MIRO and ZRS in a high-mobility 2DHG hosted in a pure Ge/SiGe quantum well. According to this theory, when a Hall bar is illuminated, the orbit centers of the Landau states perform a classical trajectory consisting in a harmonic motion along the direction of the current. Thus, the 2D carriers move in phase and harmonically at the radiation frequency altering dramatically the scattering conditions and giving rise eventually to MIRO and, at higher P, ZRS.

Working with the valence band gives us a new scenario as is the possibility of having two different carriers sustaining the current and being coupled simultaneously with radiation. We expect that this situation will deeply change the MIRO profile. They would be light and heavy holes being driven by MW and giving rise to a clear interference regime being evidenced in the calculated R_{xx}. In the same way, the interplay of lower temperatures (T) and higher P can reveal two different resonance peaks at different B in R_{xx}, each one corresponding to heavy and light holes. Finally, we have studied the hole-based MIRO dependence on T and P obtaining similar results as with electrons. For instance, the calculated dependence on P follows a sublinear power law that has been

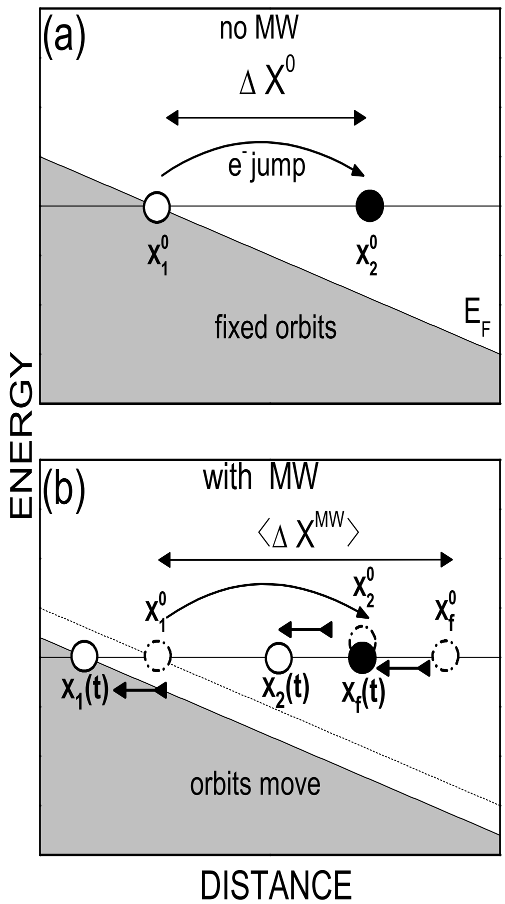

FIG. 1: Schematic diagrams for magnetotransport without and with MW. In (a) no MW field is present and due to impurity scattering (elastic), carriers jump between fixed-position orbits or Landau states. The positions are fixed by the center of the respective orbits, X_1^0 and X_2^0. The average advanced distance is given by the difference of the orbits center positions, $\Delta X^0 = X_2^0 - X_1^0$. In (b) orbits move backwards during the scattering jump and on average carriers advance further than in the no-MW case. This corresponds to a MIRO peak (see text below). Now the average advanced distance is given by $\langle \Delta X^{MW} \rangle$.

already obtained in previous experiments[16] with electrons and theoretically confirmed[29,30]. As expected the corresponding exponent of the power law is around 0.5.

II. THEORETICAL MODEL

The *radiation-driven electron orbits model*, was developed to explain the magnetoresistance response to radiation of a 2DEG at low B[18,26–28]. We first obtain an exact expression of the electronic wave vector for a 2DES in a perpendicular B and radiation. Thus, the total hamiltonian H can be written as:

$$\begin{aligned} H &= \frac{P_x^2}{2m^*} + \frac{1}{2} m^* w_c^2 (x - X)^2 - eE_{dc}X + \\ &+ \frac{1}{2} m^* \frac{E_{dc}^2}{B^2} - eE_0 \cos wt (x - X) - \\ &- eE_0 \cos wt X \\ &= H_1 - eE_0 \cos wt X \end{aligned} \tag{1}$$

X is the center of the orbit for the electron spiral motion and dependent on B and E_{dc} which is the DC electric field in the x direction. E_0 is the intensity for the MW field and H_1 is the hamiltonian corresponding to a forced harmonic oscillator whose orbit is centered at X. H_1 can be solved exactly[27,28] making possible an exact solution for the total wave function of H[18,26–29]:

$$\Psi_N(x,t) \propto \phi_n(x - X - x_{cl}(t), t) \tag{2}$$

where ϕ_n is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator. $x_{cl}(t)$ is the classical solution of a forced and damped harmonic oscillator:

$$x_{cl}(t) = \frac{eE_o}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \cos wt = A \cos wt \tag{3}$$

where γ is a phenomenologically-introduced damping factor for the interaction of carriers with acoustic phonons. w is the MW angular frequency where $w = 2\pi f$, f being the frequency. Since this model can be applied equally either to electrons or holes, we will refer to them as *carriers* for the rest of the paper.

Then, the obtained wave function is the same as the standard harmonic oscillator where the center is displaced by $x_{cl}(t)$. Thus, the carriers orbit centers are not fixed, but they oscillate harmonically at the MW frequency. This $radiation-driven$ behavior will dramatically affect the charged impurity scattering and eventually the conductivity. Next, we introduce the scattering suffered by the carriers due to charged impurities. If the scattering is weak, we can apply time dependent first order perturbation theory. First, we calculate the impurity scattering rate, W_I, between two *oscillating* Landau states, the initial $\Psi_n(t)$ and the final state $\Psi_m(t)$[18,26,31].

Secondly, and in order to calculate the drift velocity, we find the average effective distance advanced by the carrier in every scattering jump[18,26,31], ΔX^{MW}. Without radiation, one carrier in an initial Landau state Ψ_1 in an orbit center position X_1^0, undergoes a scattering process and jumps to a final Landau state Ψ_2 with an orbit center position X_2^0. On average the carrier orbits center moves in the x direction a distance

given by the difference of the two orbits center positions, $\Delta X^0 = X_2^0 - X_1^0$ (see Fig. 1(a)). With radiation, the carriers orbit center position changes in the x direction harmonically with time and is given according to our model by $X^{MW}(t) = X^0 + A\cos(wt - \theta)$. θ is a general phase being calculated applying the initial conditions, i.e., for $t = 0$, $X^{MW}(0) = X^0$, then $\theta = \pi/2 \Rightarrow X^{MW}(t) = X^0 + A\sin wt$. In other words, due to the MW field all the carrier orbit centers oscillate in phase back and forth in the x direction with $A\sin wt$. On the other hand, after the MW is on, in a given time the carrier will undergo a scattering event. If this happens when the orbits, driven by MW, move backwards we have the situation depicted in Fig. 1(b). This corresponds to an increase in the average distance advanced by the carriers giving rise to a MIRO peak. If the orbits move forwards, we will have the opposite situation, a drop in the average advanced distance, producing a MIRO valley.

If at the moment of scattering the carrier is in the *oscillating* Landau state $\Psi_1(t)$ at the position $X_1(t) = X_1^0 + A\sin wt$, after a time τ, what we call *flight time*, it will reach a *final oscillating* Landau state $\Psi_f(t+\tau)$ located in the position given by $X_f(t+\tau) = X_f^0 + A\sin w(t+\tau)$. In general this position is not longer occupied by Ψ_2 since its position, $X_2(t)$, has been shifted a certain τ-dependent distance and it is now being taken by Ψ_f. Then, the advanced distance under MW is:

$$\begin{aligned} \Delta X^{MW} &= X_f(t+\tau) - X_1(t) \\ &= X_f^0 + A\sin w(t+\tau) - X_1^0 - A\sin wt \end{aligned} \tag{4}$$

The *flight time* τ, is strictly the time it takes the carrier to scatter from one orbit to another. This time is part of the scattering time, τ_s, that is normally defined as the average time between scattering events. Therefore the scattering time would be made up of the time flight plus the time the carrier lies in the new orbit till the next scattering event takes place.

Now considering a stationary regime, i.e., averaging in time, we obtain:

$$\langle \Delta X^{MW} \rangle = X_f^0 - X_1^0 \tag{5}$$

We can express $\langle \Delta X^{MW} \rangle$ with respect to the average advanced distance in the dark ΔX^0:

$$\langle \Delta X^{MW} \rangle = \Delta X^0 + (X_f^0 - X_2^0) \tag{6}$$

where the *distance shift*, $(X_f^0 - X_2^0)$, is going to be the term responsible of the MW driven R_{xx} oscillations or MIRO. Therefore, according to that expression if $X_f^0 > X_2^0$ we will have a larger advanced distance in the x direction producing a peak in the conductivity and in turn in R_{xx}. In the opposite situation if $X_f^0 < X_2^0$ we would obtain a valley in R_{xx} with respect to the dark scenario. When $X_f^0 = X_2^0$ we would obtain a node, where R_{xx} with radiation is equal to the dark R_{xx}, and finally the most striking situation happens when $X_f^0 < X_1^0$ where ZRS occur.

To calculate the *distance shift* we have to take into account, as we said above, that the position occupied by the orbit Ψ_2 at a given time t will be taken by the orbit Ψ_f after a time $t+\tau$: $X_2(t) = X_f(t+\tau)$. On the other hand, mid-positions of the orbits are obtained when when $wt = 2\pi n$, n being a positive integer, and substituting this condition in the last equation: $X_2^0 = X_f^0 + A\sin w\tau$ We finally obtain an expression for the average *distance shift*:

$$(X_f^0 - X_2^0) = -A\sin w\tau \tag{7}$$

Substituting in Eq. 6 we can write for $\langle \Delta X^{MW} \rangle$:

$$\langle \Delta X^{MW} \rangle \;=\; \Delta X^0 - A\sin w\tau \tag{8}$$

Thus in principle, the two key variables to observe MIRO and ZRS are w and τ. For instance, if for a fixed w, τ is very small then $\sin w\tau \to 0$, then, there will be no effect on R_{xx} and we would obtain no MIRO. On the other hand, if τ is much larger than T, ($\tau >> T = \frac{2\pi}{w}$), T being the period of MW, then the average value of $\sin w\tau$ would be zero and no MIRO will be observed either. Therefore we can conclude that in order to observe MIRO, τ has to be of the order of the period of MW, otherwise the effect would vanish.

The longitudinal conductivity σ_{xx} is given by[31]

$$\sigma_{xx} \propto \int dE \frac{\langle \Delta X^{MW} \rangle}{\tau_s} \tag{9}$$

being E the energy. To obtain R_{xx} we use the relation $R_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$, where $\sigma_{xy} \simeq \frac{p_i e}{B}$, p_i being the holes density, and $\sigma_{xx} \ll \sigma_{xy}$. Thus,

$$R_{xx} \propto -A\sin w\tau \tag{10}$$

From this expression the MIRO minima positions fulfill the condition:

$$w\tau = \frac{\pi}{2} + 2\pi n \Rightarrow w = \frac{2\pi}{\tau}\left(\frac{1}{4} + n\right) \tag{11}$$

n being a positive integer. And for the MIRO maxima:

$$w\tau = \frac{3\pi}{2} + 2\pi n \Rightarrow w = \frac{2\pi}{\tau}\left(\frac{3}{4} + n\right) \tag{12}$$

We can obtain also expressions for the MIRO nodes, or the points where the radiation curve crosses the dark curve. If we take as a reference any MIRO peak, the right node fulfills, $w = \frac{2\pi}{\tau}(n - 1/2)$. And the left one, $w = \frac{2\pi}{\tau} n$. To have an evaluation and a deeper physical meaning of the *flight time* τ we can first use a quantum mechanical approach using the time-energy uncertainty relation[32]: $\Delta t \cdot \Delta E \simeq h$. In our case, τ is the time it takes the carrier to evolve from the initial Landau state Ψ_n to the final one Ψ_m, so the uncertainty of time is represented by

τ: $\Delta t = \tau$. During this time (flight between orbits), the state function of the carrier can be assumed as a linear superposition of the two Landau states involved in the process, and with respective energies: E_n and E_m. If we measured the energy we would obtain either E_n or E_m, then the uncertainty of the energy is: $\Delta E = |E_n - E_m| = \hbar w_c$. For the last expression we have assumed that $m = n+1$. This is a reasonable assumption because if the carrier jumps from the Landau state with index n, the closest in energy Landau state, with index $n+1$, is the most likely state for the carrier to end up after the scattering event. Therefore the uncertainty relation reads:

$$\Delta t \cdot \Delta E = \tau \cdot \hbar w_c \simeq h \Rightarrow \tau \simeq \frac{2\pi}{w_c} = T_c \qquad (13)$$

where T_c is the cyclotron period: the carrier *flight time* τ, turns out to be approximately equal to the cyclotron period.

The semiclassical assessment of τ would consist in the next: during the scattering jump from one orbit to another, in a time τ, the carriers in their orbits complete one full loop, which implies that $\tau = T_c$. Therefore the carrier involved in the scattering ends up in the same relative position inside the final orbit as the one it started from in the initial one. The reason for this is that the dynamics of the orbits (Landau states) is governed on average by the position of the center of the orbit irrespective of the carrier position inside the orbit when the scattering takes place. Then, on average, both initial and final semiclassical positions are identical in their respective orbits. Then, during the *flight time*, the carriers in their orbits complete one loop and then $\tau = T_c$. If next, we substitute the result $\tau = \frac{2\pi}{w_c}$ in the MIRO extrema expressions, we obtain:

$$\frac{w}{w_c} = \left(\frac{1}{4} + n\right) \qquad (14)$$

for the minima and,

$$\frac{w}{w_c} = \left(\frac{3}{4} + n\right) \qquad (15)$$

for the maxima. The above expressions are exactly the same as the ones experimentally obtained previously by Mani et al[4,7]. Therefore, we can conclude, based on our theory, that the physical origin of the 1/4-cycle phase shift in MIRO has to do with the harmonically swinging nature of the irradiated Landau states and the elastic scattering between them. Radiation frequency and carrier flight time are the key variables ruling the process.

Nevertheless, the most important result turns out to be the final expression of irradiated magnetoresistance where the part responsible of MIRO can be written as:

$$R_{xx} \propto -A \sin\left(2\pi \frac{w}{w_c}\right) \qquad (16)$$

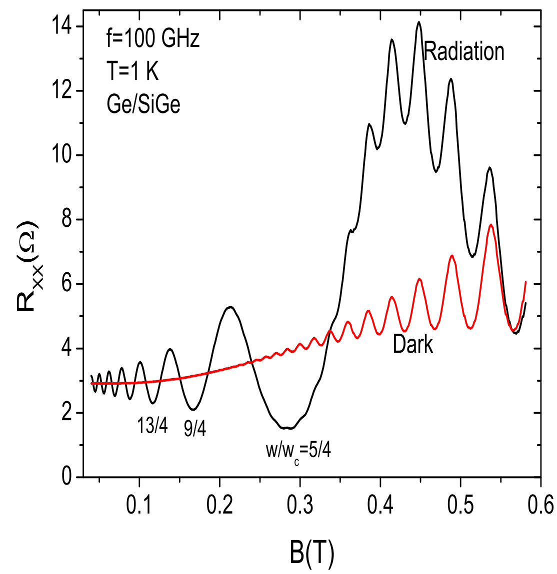

FIG. 2: Calculated magnetoresistance, R_{xx}, vs magnetic field, B, for dark and radiation of frequency $f = w/2\pi = 100$ GHz and temperature $T = 1.0K$. For the latter we obtain the radiation-induced resistance oscillations characterized by a series of peaks and valleys in function of B and the radiation frequency w. The R_{xx} oscillations are calculated for a 2DHG in a Ge/SiGe quantum well with a hole effective mass, $m^* = 0.1m_0$ where m_0 is the free electron mass. Minima positions are indicated by $w/w_c = 5/4, 9/4, 13/4$.

This result can be described as universal since it depends only on external variables such as radiation and magnetic field and it is totally independent of the type of the sample semiconductor material.

Finally, in a scenario where we had two different type of carriers simultaneously coupled to MW, as light and heavy holes, R_{xx} would be written as:

$$R_{xx} \propto -\left[A_{lh} \sin\left(2\pi \frac{w}{w_{c,lh}}\right) + A_{hh} \sin\left(2\pi \frac{w}{w_{c,hh}}\right)\right] \qquad (17)$$

where A_{lh} and $w_{c,lh}$ are the amplitude and cyclotron frequency for the light holes and A_{hh} and $w_{c,hh}$ for the heavy holes. Then, we can predict an interference profile in the standard MIRO, or at least a clear distorted profile, depending on the relative values of the light and heavy hole effective masses.

III. RESULTS

In Fig. 2, we plot calculated R_{xx} vs B for a high mobility 2DHG hosted in a pure Ge/SiGe quantum well corresponding to the cases of dark and MW of 100 GHz. The Ge quantum well is fully strained[25,36] and the hole density is $p \approx 2.8 \times 10^{11} cm^{-2}$. As a result only the

heavy hole band is available to provide holes with an effective mass of $m^* \approx 0.1m_0$[33] where m_0 is the free electron mass. For the MW curve we obtain two oscillatory structures, one corresponds to MIRO characterized by a series of peaks and valleys in function of B and w. In other words, the standard MIRO profile very similar to the ones previously obtained for electrons. The other structure corresponds to the Shubnikov-de Haas oscillations. As we said above, according to our model if the hole orbits, in their oscillating MW-driven motion, are moving backwards, on average during the scattering jump the hole advances a larger distance than in the dark case ($\langle \Delta X^{MW} \rangle > \Delta X^0$). Therefore, we will have an increasing R_{xx} and eventually a MIRO peak. On the other hand, if the orbits are moving forwards the hole will advance on average a shorter distance during the scattering, ($\langle \Delta X^{MW} \rangle < \Delta X^0$), giving rise to a decreasing R_{xx} and a MIRO valley. Minima positions are indicated by $w/w_c = 5/4, 9/4, 13/4$. As in the case of electrons, for higher P, one or more valleys could evolve into ZRS. This is what we present in Fig. 3 for the same material as in Fig. 2. We obtain a clear region of ZRS between 0.10 and 0.12 T. In the inset we present a schematic diagram explaining the physical origin of ZRS: if we increase further the MW power, we will eventually reach the situation where the orbits are moving forwards but their amplitude is larger than the scattering jump. In this case the hole jump is blocked because the final state is occupied.

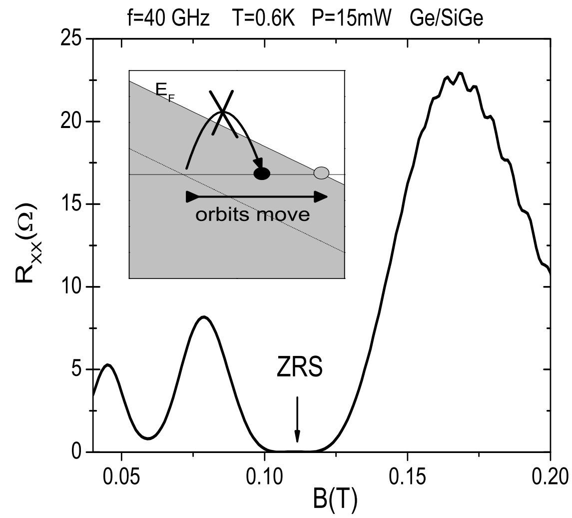

FIG. 3: Calculated R_{xx} vs B in the presence of radiation of frequency $f = 40$ GHz and $T = 0.6K$. We obtain a very clear region of ZRS between 0.10 and 0.12 T. In the inset we present a schematic diagram explaining the physical origin of ZRS. For an increasing MW power, we will eventually reach the situation where the orbits are moving forwards but their amplitude is larger than the scattering jump. In this case the hole jump is blocked because the final state is occupied.

In the hypothetical case of having a 2DHG in unstrained Ge, we would have the heavy and light hole valence bands degenerate at the Γ point. As a result, both type of holes would be available to participate in the transport and couple to MW. The theoretical outcome, according to Eq. 17, would be an interference scenario that would be evidenced in R_{xx}. This is presented in Fig 4 where we plot calculated R_{xx} vs B for unstrained Ge, MW of frequency 100 GHz and $T = 1K$. We have considered the bulk effective masses for light and heavy holes of Ge: $m^*_{HH} = 0.28m_0$ and $m^*_{LH} = 0.044m_0$. As expected, we obtain a very clear interference profile for MIRO. For a more realistic scenario we have considered the case of a $100\AA$ GaAs/GaAlAs quantum well[37]. For this platform it is possible, applying a uniaxial compressive stress, to shift downwards the highest heavy hole band, making it degenerate with light hole band at the Γ point[37]. The corresponding calculated effective masses turn out to be: $m^*_{HH} = 0.38m_0$ and $m^*_{LH} = 0.09m_0$[37]. In Fig. 5 we present calculated R_{xx} vs B for this case. The MW frequency is 100 GHz and $T = 1K$. We obtain again a distorted profile for MIRO, revealing an apparent interference effect.

In Fig. 6 we present similar case as in Fig. 5 but now the MW frequency is 50 GHz and $T = 0.2K$. We have lowered the temperature in order to weaken the damping γ, (see expression of A) and obtain the corresponding resonance peaks of light and heavy holes. The former is observed at $B \simeq 0.2T$ and the latter at $B \simeq 0.7T$. Interestingly, from the B-position of these peaks we could obtain simultaneously the effective masses of carriers involved in the magnetotransport. The heavier the carrier the more displaced the peak to higher B.

In Fig. 7a we present P-dependence of irradiated R_{xx} vs B for $f = 40$ GHz and $T = 0.6$ K for a 2DHG in a Ge/SiGe quantum well. We sweep P from dark to $P = 10.7$ mW. We observe that the radiation-induced oscillations get larger as P increases, showing a similar behavior as in MIRO with electrons. In Fig. 7b, we present $\Delta R_{xx} = R^{MW}_{xx} - R^0_{xx}$ vs P, for data coming from the main R_{xx} peak. R^0_{xx} is the magnetoresistance for dark and R^{MW}_{xx} when the radiation field is on. We fit the data obtaining a sublinear P-dependence:

$$\Delta R_{xx} \propto P^{\alpha} \tag{18}$$

where $\alpha \approx 0.5$ and it is straightforwardly explained with our model in terms of:

$$E_0 \propto \sqrt{P} \Rightarrow \Delta R_{xx} \propto \sqrt{P} \tag{19}$$

and in agreement with previous experimental[16] and theoretical[30] results obtained for electrons.

In Fig 8a, we present the T-dependence of irradiated R_{xx} vs B for a MW frequency $f = 100$ GHz and same material as in Fig. 7. We sweep the temperature from $T = 1.0$ K to $T = 5.0$ K. We observe a clear decrease of the oscillation amplitude for increasing T. Eventually a R_{xx} response similar to dark is reached, but without the Shubnikov-de Haas oscillations that are very affected by

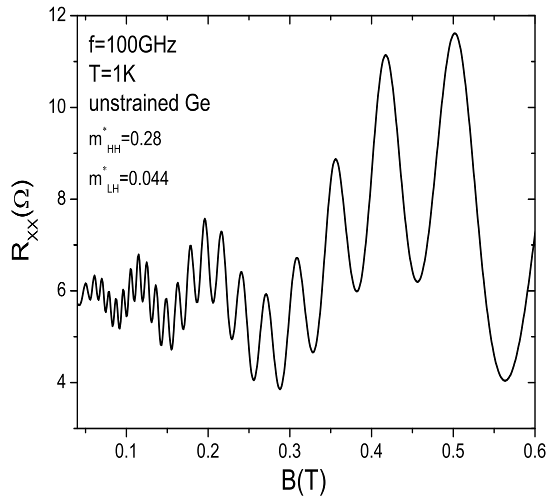

FIG. 4: Calculated R_{xx} vs B in the presence of radiation of frequency $f = 100\ GHz$ and $T = 1K$. We plot the hypothetical case of a 2DHG in unstrained Ge. In this case both heavy and light hole valence band are degenerate at the Γ point. With this scenario we have available both types of holes to couple to MW and take part in the magnetotransport. We observe a modulated MIRO profile as a result of the interference regime produced by the presence of two different type of carriers. We have considered the bulk effective masses for Ge: $m^*_{HH} = 0.28m_0$ and $m^*_{LH} = 0.044m_0$.

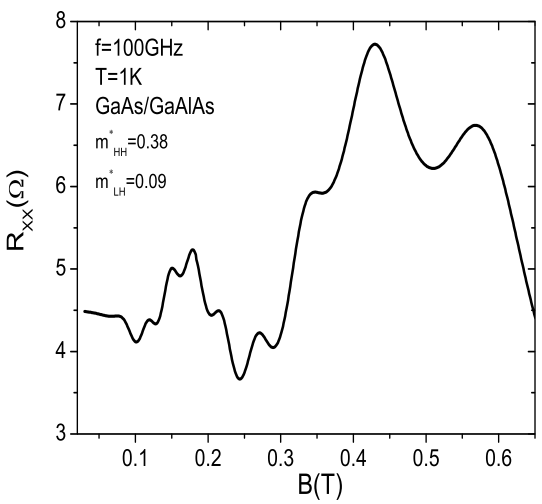

FIG. 5: Calculated R_{xx} vs B in the presence of radiation of frequency $f = 100$ GHz and $T = 1K$ for a 100 Å GaAs/GaAlAs quantum well. For this case it is possible applying a uniaxial stress to shift downwards the highest heavy hole band, making it degenerate with light hole band at the Γ point. We obtain an interference profile for MIRO.

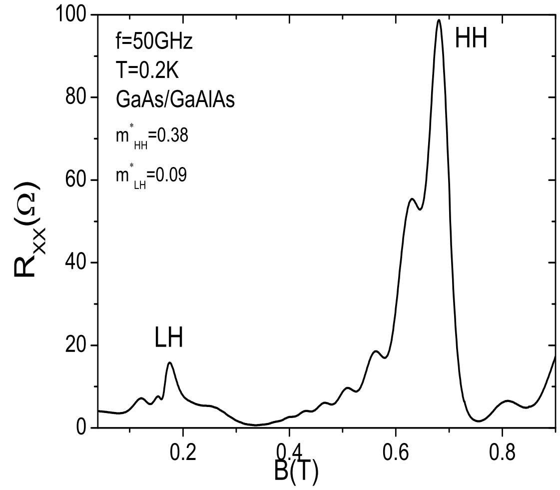

FIG. 6: Same as in Fig. 5 but for a MW frequency $f = 50$ GHz and $T = 0.2$ K. The light hole peak is observed at $B \simeq 0.2T$ and the heavy hole one at $B \simeq 0.7T$.

increasing T making them to vanish. The T-dependence, according to the model, is explained with the damping parameter γ which represents the interaction of carriers with the lattice ions giving rise to the emission of acoustic phonons. This interaction can be calculated in terms of the scattering rate of carriers with longitudinal acoustic phonons through the Fermi's golden rule[38,39]. Thus, γ turns out to be linear with T and then an increasing T means an increasing γ and smaller R_{xx} oscillations. When the damping is strong enough (higher T) R_{xx} oscillations collapse. In Fig. 8b, we present ΔR_{xx}, of the peak labeled in the upper panel with $+2$ and the valley labeled with -1 vs T^{-2}. The two curves are vertically shifted for clarity. We observe that for hight T, ΔR_{xx} is approximately linear with T^{-2} and for low T, ΔR_{xx} falls below the linear dependence approaching a constant value, i. e., independent of T^{-2}. We can find explanation considering the expressions obtained from the model:

$$\Delta R_{xx} \propto \frac{eE_o}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \tag{20}$$

and $\gamma \propto T$.

Accordingly, with high T, $\gamma^4 > (w_c^2 - w^2)^2$ and then we can approximate $\Delta R_{xx} \propto T^{-2}$ giving a linear dependence. Yet, for low T, $\gamma^4 < (w_c^2 - w^2)^2$ making ΔR_{xx} independent of T and approaching a horizontal line as plotted in Fig. 8b. To confirm this last result we have calculated R_{xx} for much lower T reaching 50 mK. We obtain that ΔR_{xx} tends clearly to a constant value. We present these results in Fig 9. where we plot ΔR_{xx} versus T. Here we sweep T from 50 mK to 5K. According to out theory, when T and in turn γ tend to 0, ΔR_{xx} tends to a constant value, when w_c is far from resonance,

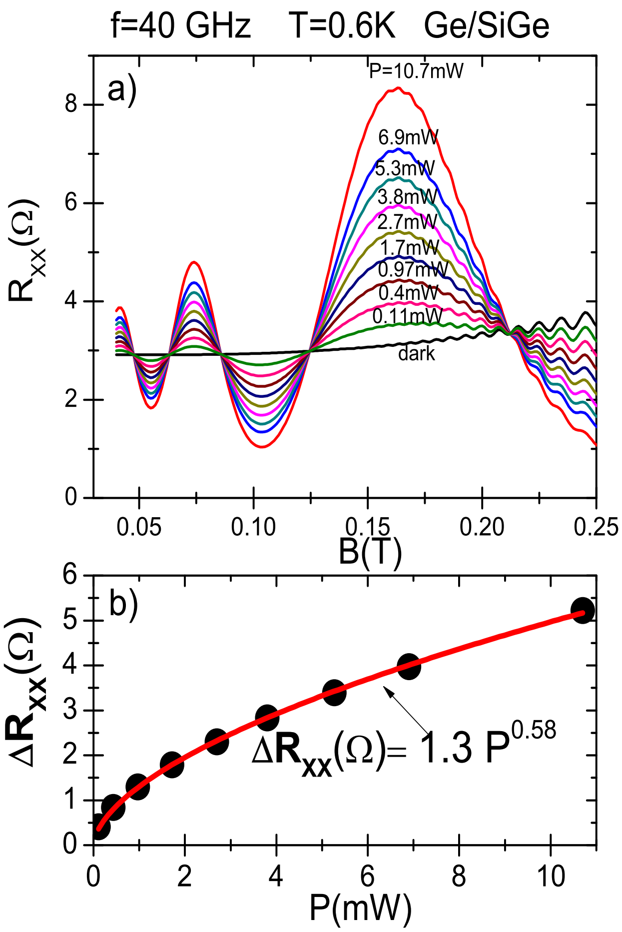

FIG. 7: a) Calculated P-dependence of R_{xx} vs B for $f = 40$ GHz and $T = 0.6$ K for a 2DHG in a Ge/SiGe quantum well. We sweep the radiation power P from dark to $P = 10.7$ mW. We observe that the radiation-induced oscillation increases giving rise to larger peaks and deeper valleys. b) Calculated amplitude $\Delta R_{xx} = R_{xx}^{rad} - R_{xx}^{0}$ vs P, for data coming from the main peak. R_{xx}^{0} is the magnetoresistance for dark and R_{xx}^{rad} when the radiation field is on. We fit the data obtaining a sublinear P-dependence where the exponent is close to 0.5.

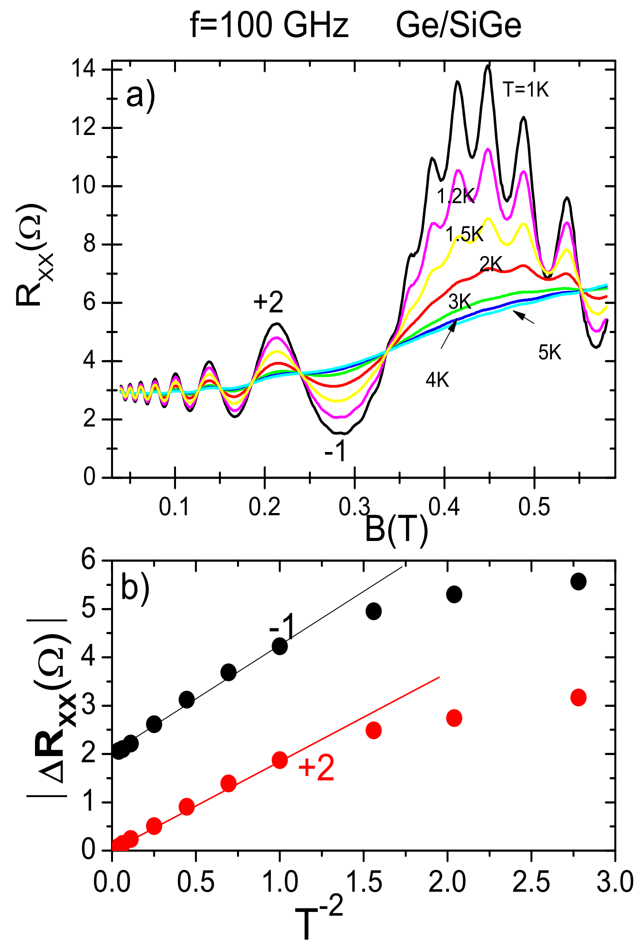

FIG. 8: a) Calculated T-dependence of R_{xx} vs B for $f = 100$ GHz of a 2DGH in a Ge/SiGe quantum well. We sweep the temperature from $T = 1$ K to $T = 5$ K. We observe a clear decrease of the oscillation for increasing T, eventually reaching a R_{xx} response similar to dark, but without the Shubnikov-de Haas oscillations that are very affected by increasing T. b) Absolute values of R_{xx} amplitudes, ΔR_{xx}, of the labeled peak $+2$ and labeled valley -1 vs T^2. The two curves are vertically shifted for clarity.

as can be obtained from equation [20]. This is simply a prediction, that can be applies also to electrons, from our theoretical model because experiments on MIRO studying T-dependence have not reached so low temperatures to date. Therefore, the real MIRO behavior at such very-low-temperature could serve to discriminate among the existing theories and give credibility to the ones predicting similar results as experiments.

IV. CONCLUSIONS

In summary, we have presented a theoretical study on MIRO in a 2DHG hosted in a Ge/SiGe quantum well in order to demonstrate that MIRO and zero resistance states are universal effects. We obtain calculated R_{xx} revealing MIRO and ZRS. We have analytically deduced a universal expression for the irradiated magnetoresistance, explaining the origin of the minima positions and their 1/4 cycle phase shift. The outcome is that these phenomena are universal and only depend on radiation and cyclotron frequencies. On the other hand, they turn

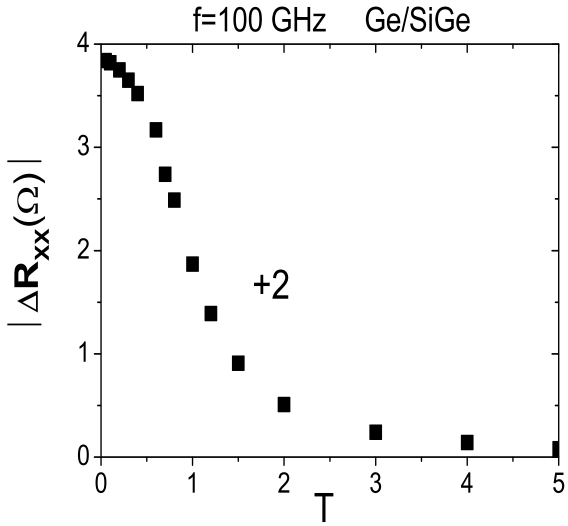

FIG. 9: Calculated T-dependence of absolute values of R_{xx} amplitudes, ΔR_{xx} vs T for $f = 100$ GHz of a 2DGH in a Ge/SiGe quantum well. We plot the data corresponding to MIRO peak labelled +2. We sweep the temperature from $T = 50$ mK to $T = 5$ K. We observe that ΔR_{xx} tend to a constant value as T tends to 0.

out to be independent of the type of semiconductor material. Interestingly, we study the possibility of having simultaneously two different carriers driven by radiation: light and heavy holes. As a result the calculated magnetoresistance reveals an interference regime due to the different effective masses of the two types of carriers. In the same way, we obtain two different resonance peaks at low enough temperature, corresponding to the two carriers. Finally, we study the dependence on microwave power and temperature obtaining a similar behaviour as with electrons. In the power dependence we obtain a sublinear law which relates the amplitude of the resistance oscillations and the applied power, being the exponent approximately equal to 0.5. For the temperature dependence we obtain, as expected, a vanishing effect on the radiation-induced resistance oscillations for increasing temperature. Interestingly, we also obtain that the amplitude of MIRO tends to a constant values as temperatures tends to 0.

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